Appendix B

Mathematical Basics

Squares and Other Powers

An exponent, or a power, is mathematical shorthand for repeated multiplications. For example, the exponent "2" means to multiply the base for that exponent by itself (in the example here, the base is "5"):

$$5^2 = 5 \times 5 = 25$$

The exponent is "2" and the base is the number "5." This expression (multiplying a number by itself) is also called a square. Any number raised to the power of 2 is being squared. Any number raised to the power of 3 is being cubed:

$$5^3 = 5 \times 5 \times 5 = 125$$

A number raised to the fourth power is equal to that number multiplied by itself four times, and so on for higher powers. In general:

$$n^x = n \times n^{x-1}$$

Calculating Percents

A percent is a way of expressing a fractional amount of something using a whole divided into 100 parts. A percent is a ratio whose denominator is 100. We use the percent symbol, %, to show percent. Thus, 25% means a ratio of $\frac{25}{100}$, 3% means a ratio of $\frac{3}{100}$, and 100 % percent means $\frac{100}{100}$, or a whole.

Converting Percents

A percent can be converted to a fraction by writing the value of the percent as a fraction with a denominator of 100 and simplifying the fraction if possible.

$$25\% = \frac{25}{100} = \frac{1}{4}$$

A percent can be converted to a decimal by writing the value of the percent as a fraction with a denominator of 100 and dividing the numerator by the denominator.

$$10\% = \frac{10}{100} = 0.10$$

To convert a decimal to a percent, write the decimal as a fraction. If the denominator of the fraction is not 100, convert it to a fraction with a denominator of 100, and then write the fraction as a percent.

$$0.833 = \frac{833}{1000} = \frac{83.3}{100} = 83.3\%$$

To convert a fraction to a percent, first convert the fraction to a decimal, and then convert the decimal to a percent.

$$\frac{3}{4} = 0.75 = \frac{75}{100} = 75\%$$

Suppose a researcher finds that 15 out of 23 students in a class are carriers of *Neisseria meningitides*. What percentage of students are carriers? To find this value, first express the numbers as a fraction.

$$\frac{\text{carriers}}{\text{total students}} = \frac{15}{23}$$

Then divide the numerator by the denominator.

$$\frac{15}{23} = 15 \div 23 \approx 0.65$$

Finally, to convert a decimal to a percent, multiply by 100.

$$0.65 \times 100 = 65\%$$

The percent of students who are carriers is 65%.

You might also get data on occurrence and non-occurrence; for example, in a sample of students, 9 tested positive for *Toxoplasma* antibodies, while 28 tested negative. What is the percentage of seropositive students? The first step is to determine the "whole," of which the positive students are a part. To do this, sum the positive and negative tests.

$$\text{positive} + \text{negative} = 9 + 28 = 37$$

The whole sample consisted of 37 students. The fraction of positives is:

$$\frac{\text{positive}}{\text{total students}} = \frac{9}{37}$$

To find the percent of students who are carriers, divide the numerator by the denominator and multiply by 100.

$$\frac{9}{37} = 9 \div 37 \approx 0.24$$
$$0.24 \times 100 = 24\%$$

The percent of positive students is about 24%.

Another way to think about calculating a percent is to set up equivalent fractions, one of which is a fraction with 100 as the denominator, and cross-multiply. The previous example would be expressed as:

$$\frac{9}{37} = \frac{x}{100}$$

Now, cross multiply and solve for the unknown:

$$
\begin{aligned}
9 \times 100 &= 37x \\
\frac{9 \times 100}{37} &= x && \text{Divide both sides by 37} \\
\frac{900}{37} &= x && \text{Multiply} \\
24 &\approx x && \text{Divide}
\end{aligned}
$$

The answer, rounded, is the same.

Multiplying and Dividing by Tens

In many fields, especially in the sciences, it is common to multiply decimals by powers of 10. Let's see what happens when we multiply 1.9436 by some powers of 10.

$$
\begin{aligned}
1.9436(10) &= 19.436 \\
1.9436(100) &= 194.36 \\
1.9436(1000) &= 1943.6
\end{aligned}
$$

The number of places that the decimal point moves is the same as the number of zeros in the power of ten. **Table B1** summarizes the results.

Multiply by	Zeros	Decimal point moves . . .
10	1	1 place to the right
100	2	2 places to the right
1,000	3	3 places to the right
10,000	4	4 places to the right

Table B1

We can use this pattern as a shortcut to multiply by powers of ten instead of multiplying using the vertical format. We can count the zeros in the power of 10 and then move the decimal point that same number of places to the right.

So, for example, to multiply 45.86 by 100, move the decimal point 2 places to the right.

$$45.86 \times 100 = 4586.$$

Sometimes when we need to move the decimal point, there are not enough decimal places. In that case, we use zeros as placeholders. For example, let's multiply 2.4 by 100. We need to move the decimal point 2 places to the right. Since there is only one digit to the right of the decimal point, we must write a 0 in the hundredths place.

$$2.4 \times 100 = 240.$$

When dividing by powers of 10, simply take the opposite approach and move the decimal to the left by the number of zeros in the power of ten.

Let's see what happens when we divide 1.9436 by some powers of 10.

$$1.9436 \div 10 \ = \ 0.19436$$
$$1.9436 \div 100 \ = \ 0.019436$$
$$1.9436 \div 1000 \ = \ 0.0019436$$

If there are insufficient digits to move the decimal, add zeroes to create places.

Scientific Notation

Scientific notation is used to express very large and very small numbers as a product of two numbers. The first number of the product, the digit term, is usually a number not less than 1 and not greater than 10. The second number of the product, the exponential term, is written as 10 with an exponent. Some examples of scientific notation are given in **Table B2**.

Standard Notation	Scientific Notation
1000	1×10^3
100	1×10^2
10	1×10^1
1	1×10^0
0.1	1×10^{-1}
0.01	1×10^{-2}

Table B2

Scientific notation is particularly useful notation for very large and very small numbers, such as $1{,}230{,}000{,}000 = 1.23 \times 10^9$, and $0.00000000036 = 3.6 \times 10^{-10}$.

Expressing Numbers in Scientific Notation

Converting any number to scientific notation is straightforward. Count the number of places needed to move the decimal next to the left-most non-zero digit: that is, to make the number between 1 and 10. Then multiply that number by 10 raised to the number of places you moved the decimal. The exponent is positive if you moved the decimal to the left and negative if you moved the decimal to the right. So

$$2386 = 2.386 \times 1000 = 2.386 \times 10^3$$

and

$$0.123 = 1.23 \times 0.1 = 1.23 \times 10^{-1}$$

The power (exponent) of 10 is equal to the number of places the decimal is shifted.

Logarithms

The common logarithm (log) of a number is the power to which 10 must be raised to equal that number. For example, the common logarithm of 100 is 2, because 10 must be raised to the second power to equal 100. Additional examples are in **Table B3**.

Number	Exponential Form	Common Logarithm
1000	10^3	3
10	10^1	1
1	10^0	0
0.1	10^{-1}	−1
0.001	10^{-3}	−3

Table B3

To find the common logarithm of most numbers, you will need to use the LOG button on a calculator.

Rounding and Significant Digits

In reporting numerical data obtained via measurements, we use only as many significant figures as the accuracy of the measurement warrants. For example, suppose a microbiologist using an automated cell counter determines that there are 525,341 bacterial cells in a one-liter sample of river water. However, she records the concentration as 525,000 cells per liter and uses this rounded number to estimate the number of cells that would likely be found in 10 liters of river water. In this instance, the last three digits of the measured quantity are not considered *significant.* They are rounded to account for variations in the number of cells that would likely occur if more samples were measured.

The importance of significant figures lies in their application to fundamental computation. In addition and subtraction, the sum or difference should contain as many digits to the right of the decimal as that in the *least* certain (indicated by underscoring in the following example) of the numbers used in the computation.

Suppose a microbiologist wishes to calculate the total mass of two samples of agar.

$$4.38\underline{3} \text{ g}$$
$$\underline{3.002}\underline{1} \text{ g}$$
$$7.38\underline{5} \text{ g}$$

The least certain of the two masses has three decimal places, so the sum must have three decimal places.

In multiplication and division, the product or quotient should contain no more digits than than in the factor containing the *least* number of significant figures. Suppose the microbiologist would like to calculate how much of a reagent would be present in 6.6 mL if the concentration is 0.638 g/mL.

$$0.63\underline{8} \; \frac{\text{g}}{\text{mL}} \times 6.\underline{6} \text{ mL} = 4.1 \text{ g}$$

Again, the answer has only one decimal place because this is the accuracy of the least accurate number in the calculation.

When rounding numbers, increase the retained digit by 1 if it is followed by a number larger than 5 ("round up"). Do

not change the retained digit if the digits that follow are less than 5 ("round down"). If the retained digit is followed by 5, round up if the retained digit is odd, or round down if it is even (after rounding, the retained digit will thus always be even).

Generation Time

It is possible to write an equation to calculate the cell numbers at any time if the number of starting cells and doubling time are known, as long as the cells are dividing at a constant rate. We define N_0 as the starting number of bacteria, the number at time $t = 0$. N_i is the number of bacteria at time $t = i$, an arbitrary time in the future. Finally we will set j equal to the number of generations, or the number of times the cell population doubles during the time interval. Then we have,

$$N_i = N_0 \times 2^j$$

This equation is an expression of growth by binary fission.

In our example, $N_0 = 4$, the number of generations, j, is equal to 3 after 90 minutes because the generation time is 30 minutes. The number of cells can be estimated from the following equation:

$$
\begin{aligned}
N_i &= N_0 \times 2^j \\
N_{90} &= 4 \times 2^3 \\
N_{90} &= 4 \times 8 = 32
\end{aligned}
$$

The number of cells after 90 minutes is 32.

Most Probable Number

The table in **Figure B1** contains values used to calculate the most probable number example given in **How Microbes Grow**.

Most Probable Number Table					
Number of tubes giving a positive reaction for a 5-tube set			MPN (per 100 ml)	95% Confidence Limits	
10 ml	1 ml	0.1 ml		Low	High
0	0	0	<2	<1	7
0	1	0	2	<1	7
0	2	0	4	<1	11
1	0	0	2	<1	7
1	0	1	4	<1	11
1	1	0	4	<1	11
1	1	1	6	<1	15
2	0	0	5	<1	13
2	0	1	7	1	17
2	1	0	7	1	17
2	1	1	9	2	21
2	2	0	9	2	21
2	3	0	12	3	28
3	0	0	8	1	19
3	0	1	11	2	25
3	1	0	11	2	25
3	1	1	14	4	34
3	2	0	14	4	34
3	2	1	17	5	46
3	3	0	17	5	46
4	0	0	13	3	31
4	0	1	17	5	46
4	1	0	17	5	46
4	1	1	21	7	63
4	1	2	26	9	78
4	2	0	22	7	67
4	2	1	26	9	78
4	3	0	27	9	80
4	3	1	33	11	93
4	4	0	34	12	93
5	0	0	23	7	70
5	0	1	31	11	89
5	0	2	43	15	110
5	1	0	33	11	93
5	1	1	46	16	120
5	1	2	63	21	150
5	2	0	49	17	130
5	2	1	70	23	170
5	2	2	94	28	220
5	3	0	79	25	190
5	3	1	110	31	250
5	3	2	140	37	340
5	3	3	180	44	500

Figure B1